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Effects of Quenching and Partial Quenching on QCD Penguin Matrix Elements

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We point out that chiral transformation properties of penguin operators change in the transition from unquenched to (partially) quenched QCD. The way in which this affects the lattice determination of weak matrix elements can be understood in the framework of (partially) quenched chiral perturbation theory.

1. (Partially) Quenched QCD penguins

We consider $\Delta S = 1$ LR operators of the form

$$Q^{QCD}_{penguin} = (\overline{s}d)_L(\overline{q}q)_R \ , \tag{1}$$

with $q = u, d, s$, $(\overline{q}_1 q_2)_{L,R} = \overline{q}_1\gamma_\mu P_{L,R} q_2$, and $P_{L,R} = (1 \mp \gamma_5)/2$ left- and right-handed projectors. Color indices can be contracted in two ways, corresponding to the QCD penguins $Q_{5,6}$.

The operators in eq. (1) are obtained by the unquenched QCD evolution of the weak operator from the weak scale $\sim M_W$ down to the hadronic scale $\sim m_c$. At hadronic scales, matrix elements of these four-quark operators are usually computed on the lattice in quenched or partially quenched QCD ((P)QQCD). The key point we wish to address here is that the chiral transformation properties of QCD penguin operators change in the transition from unquenched to (P)QQCD. This has non-trivial consequences [1] which can be understood in the framework of (partially) quenched chiral perturbation theory ((P)QChPT) [3].

PQQCD can be systematically formulated in a lagrangian framework by coupling the gluons to three sets of quarks [3]: K valence quarks q_{vi} with masses $m_{v1}, m_{v2}, \ldots, m_{vK}$, N sea quarks q_{si} with masses $m_{s1}, m_{s2}, \ldots, m_{sN}$, and K ghost quarks q_{gi} with masses $m_{v1}, m_{v2}, \ldots, m_{vK}$. The ghost quarks are identical to the valence quarks, except for their statistics, which is chosen to be bosonic [4]. Quenched QCD corresponds to the special case $N = 0$ (no sea quarks). It can be shown that unquenched QCD below the charm threshold corresponds to the choice $K = N = 3$ and $m_{si} = m_{vi}$, $i = 1, \ldots, 3$ [3,5].

The total number of quarks in PQQCD is thus $2K + N$, and correspondingly, the chiral symmetry group enlarges from $SU(3)_L \times SU(3)_R$ to the graded group $SU(K+N|K)_L \times SU(K+N|K)_R$. As a consequence, the operators of eq. (1), which are singlets under $SU(3)_R$, are no longer singlets under the enlarged group $SU(K+N|K)_R$. They can be decomposed as

$$\begin{aligned} Q^{QCD}_{penguin} &= \frac{K}{N}\,\mathrm{str}\,(\Lambda\psi\overline{\psi}\gamma_\mu P_L)\,\mathrm{str}\,(\psi\overline{\psi}\gamma_\mu P_R) + \\ &\quad \mathrm{str}\,(\Lambda\psi\overline{\psi}\gamma_\mu P_L)\,\mathrm{str}\,(A\psi\overline{\psi}\gamma_\mu P_R)\ , \\ &\equiv \frac{K}{N}\,Q^{PQS}_{penguin} + Q^{PQA}_{penguin}\ , \end{aligned} \tag{2}$$

$$A = \mathrm{diag}(1-\frac{K}{N}, \ldots, 1-\frac{K}{N}, -\frac{K}{N}, \ldots, -\frac{K}{N})\ , \tag{3}$$

where the first K (valence) entries of A are equal to $1 - K/N$, and the next $N + K$ (sea and ghost) entries are equal to $-K/N$. The superscripts PQS and PQA indicate that these operators transform in the singlet and adjoint representations of $SU(K+N|K)_R$, respectively. In the quenched case the situation is special. The decomposition reads

$$\begin{aligned} Q^{QCD}_{penguin} &= \frac{1}{2}\,\mathrm{str}\,(\Lambda\psi\overline{\psi}\gamma_\mu P_L)\,\mathrm{str}\,(\psi\overline{\psi}\gamma_\mu P_R) + \\ &\quad \mathrm{str}\,(\Lambda\psi\overline{\psi}\gamma_\mu P_L)\,\mathrm{str}\,(\hat{N}\psi\overline{\psi}\gamma_\mu P_R)\ , \end{aligned}$$

*Talk given by E. Pallante at Lattice 2001

$$\equiv \quad \frac{1}{2}\, Q^{QS}_{penguin} + Q^{QNS}_{penguin}\ , \tag{4}$$

$$\hat{N} \quad = \quad \frac{1}{2}\mathrm{diag}(1,\ldots,1,-1,\ldots,-1)\ , \tag{5}$$

where the first K (valence) entries of $\hat{N}$ are equal to $\frac{1}{2}$, and the last K (ghost) entries are equal to $-\frac{1}{2}$. The first operator in the decomposition is a singlet, while the second is not, under $SU(K|K)_R$ (NS for non-singlet). However, the unit matrix has now a vanishing supertrace, while $\hat{N}$ has not, and $Q^{QNS}_{penguin}$ can mix with $Q^{QS}_{penguin}$ through penguin-like diagrams.

2. Representation of QCD penguins in PQChPT

QCD penguins transform as $(8,1)$ under the chiral symmetry group and start at order p^2 in ordinary ChPT (see *e.g.* ref. [6]). Denoting the adjoint representation of the PQ group by A, we found in the previous section that $Q^{PQS}_{penguin}$ transforms as $(A,1)$, while $Q^{PQA}_{penguin}$ transforms as (A,A) under $SU(K+N|K)_L \times SU(K+N|K)_R$. To lowest order in PQChPT and in euclidean space, these operators are represented by

$$\begin{aligned}
Q^{PQS}_{penguin} &\to -\alpha_1^{(8,1)}\mathrm{str}\,(\Lambda L_\mu L_\mu) + \alpha_2^{(8,1)}\mathrm{str}\,(\Lambda X_+)\\
Q^{PQA}_{penguin} &\to f^2\ \alpha^{(8,8)}\ \mathrm{str}\,(\Lambda\Sigma A\Sigma^\dagger)\ ,
\end{aligned} \tag{6}$$

where $L_\mu = i\Sigma\partial_\mu\Sigma^\dagger$, $X_\pm = 2B_0(\Sigma M^\dagger \pm M\Sigma^\dagger)$, with M the quark-mass matrix, B_0 the parameter B_0 of ref. [7], $\Sigma = \exp{(2i\Phi/f)}$ describing the partially-quenched Goldstone-meson multiplet, and f the bare pion-decay constant normalized such that $f_\pi = 132$ MeV. The low-energy constants (LECs) $\alpha^{(8,1)}_{1,2}$ also appear in the unquenched theory. However, the LEC $\alpha^{(8,8)}$ only appears in the PQ case, and multiplies an order p^0 operator, which is possible because A is nontrivial. It has a direct physical meaning, because the same LEC also appears in the bosonization of the EM penguin (which belongs to the same irreducible representation as $Q^{PQA}_{penguin}$). Quenched bosonization rules to leading order are given by

$$\begin{aligned}
Q^{QS}_{penguin} &\to -\alpha_{q1}^{(8,1)}\mathrm{str}\,(\Lambda L_\mu L_\mu) + \alpha_{q2}^{(8,1)}\mathrm{str}\,(\Lambda X_+)\\
Q^{QNS}_{penguin} &\to f^2\ \alpha_q^{NS}\ \mathrm{str}\,(\Lambda\Sigma \hat{N}\Sigma^\dagger)\ .
\end{aligned} \tag{7}$$

The quenched case differs from the partially quenched one in two ways. First, all ChPT LECs depend on N, and thus their quenched values are not necessarily equal to their $N \neq 0$ values. Second, in this case the operator $Q^{QNS}_{penguin}$ and the EM penguin do not belong to the same irrep and their LECs are in principle not related [1].

3. Kaon matrix elements in PQChPT

Since the new operators $Q^{PQA}_{penguin}$ in eq. (6) and $Q^{QNS}_{penguin}$ in eq. (7) are of order p^0 in ChPT (while singlet LR operators are of order p^2), they potentially lead to an enhancement relative to the unquenched case. As the simplest example, we consider $K \to \pi$ and $K \to 0$ matrix elements, to leading order in ChPT. New contributions also show up in direct $K \to \pi\pi$ matrix elements [2].

It turns out that $Q^{PQA}_{penguin}$ (and $Q^{QNS}_{penguin}$) do not contribute to any of these matrix elements (i.e. $K \to \pi$, $K \to 0$ and $K \to \pi\pi$) at order p^0, but they do in general contribute at order p^2. Since $Q^{PQS}_{penguin}$ starts at order p^2, the new contributions from $Q^{PQA}_{penguin}$ compete at the *leading* order of the chiral expansion of these matrix elements, and have to be taken into account even if one analyzes lattice results using only leading-order ChPT. For the PQ $K \to \pi$ matrix element, with degenerate valence quark masses ($M^2 = M_K^2 = M_\pi^2 = 2B_0 m_v$), we find at order p^2

$$[K^+ \to \pi^+]^{QCD}_{penguin} = \frac{4M^2}{f^2}\Bigg\{\alpha_1^{(8,1)} - \alpha_2^{(8,1)} - \frac{2}{(4\pi)^2}\left(1-\frac{K}{N}\right)(\beta_1^{(8,8)}+\beta_2^{(8,8)})\Bigg\}\ , \tag{8}$$

where the LECs $\beta^{(8,8)}_{1,2}$ appear in the bosonization of $Q^{PQA}_{penguin}$ at order p^2 [1]. The $K \to 0$ matrix element with arbitrary valence and sea quarks is

$$[K^0 \to 0]^{QCD}_{penguin} = \frac{4i}{f}\Bigg\{\Bigg(\alpha_2^{(8,1)} + \frac{2}{(4\pi)^2}\left(1-\frac{K}{N}\right)\beta_2^{(8,8)}\Bigg)(M_K^2 - M_\pi^2) + \frac{\alpha^{(8,8)}}{(4\pi)^2}\Bigg(\sum_{i\ valence} M^2_{3vi}(L(M_{3vi})-1) -$$

$$\sum_{i\ valence} M_{2vi}^2(L(M_{2vi})-1) - \sum_{i\ sea} M_{3si}^2(L(M_{3si})-1) + \sum_{i\ sea} M_{2si}^2(L(M_{2si})-1)\Bigg)\Bigg\} . \quad (9)$$

Here $L(M) = \log \frac{M^2}{\Lambda^2}$, and the result is given in the $\overline{MS}$ scheme, with Λ the running scale. M_{3si} (M_{2si}) is the mass of a meson made out of the 3rd (2nd) valence (*i.e.* the strange (down)) quark and the ith sea quark; analogously for M_{3vi} (M_{2vi}) with sea replaced by valence. At the order we are working, $M_{3si}^2 - M_{2si}^2 = M_{3vi}^2 - M_{2vi}^2 = M_K^2 - M_\pi^2$, which follows from $M_{3si}^2 = B_0(m_{v3} + m_{si})$, *etc.* These results also contain the unquenched result as a particular case, *i.e.* for $N = K = 3$ and by equating sea and valence quark masses, $m_{si} = m_{vi}$, $i = 1, \ldots, K$. Quenched ($N = 0$) results are obtained by replacing $\alpha_{1,2}^{(8,1)} \to \alpha_{q1,2}^{(8,1)}$, $\alpha^{(8,8)} \to \alpha_q^{NS}$ and $\beta_{1,2}^{(8,8)} \to \beta_{q1,2}^{NS}$ in eqs. (8,9), and by dropping all terms containing sea quarks.

We conclude that (in general) the bosonization of the new operator $Q_{penguin}^{PQA}$ contributes to a weak matrix element at leading order in ChPT with non-analytic terms of the form $M^2 \log M^2$, which are absent in the unquenched case, and with analytic contributions parameterized by new LECs $\beta_{1,2}^{(8,8)}$ ($N \neq 0$) or $\beta_{q1,2}^{NS}$ ($N = 0$).

4. Lattice strategies

To leading order in ChPT, the physical $K \to \pi\pi$ amplitude is determined by $\alpha_1^{(8,1)}$, and therefore this is the LEC one wishes to extract from the $K \to \pi$ matrix element. It is then clear that this can be done by considering only the singlet penguin, $Q_{penguin}^{PQS}$, in the PQ theory with $N = 3$ light sea quarks. This is equivalent to omitting all Wick contractions in which q and $\overline{q}$ in eq. (1) are contracted (eye graphs), except when they correspond to sea quarks.[2] Of course, since the $K \to \pi$ matrix element only gives the linear combination $\alpha_1^{(8,1)} - \alpha_2^{(8,1)}$, the $K \to 0$ matrix element of $Q_{penguin}^{PQS}$ is also needed, as usual [8].

In the case that $N \neq 3$, one can still follow a similar strategy in order to determine $\alpha_1^{(8,1)}$. However, since the LECs depend on the number of light sea quarks N, there is no reason why the result should be the same as that of the real world, which has $N = 3$. In particular, in the quenched case different strategies are possible. If one assumes that $\alpha_1^{(8,1)}(N = 0) \approx \alpha_1^{(8,1)}(N = 3)$, one should use the strategy described above. Alternatively, one may include both the singlet and non-singlet operators, also in the conversion to $K \to \pi\pi$ [2]. Since it is not known which (if any) of these strategies is a better approximation, we believe that the difference between them should be interpreted as an indication of the quenching error.

Finally, we stress that our observations also applies to LL penguin operators (in which case the new effects start at order p^4 [1,2]) and, more in general, to any weak matrix element to which penguin operators contribute, such as also non-leptonic B decays.

We would like to thank the RBC collaboration, Claude Bernard, Guido Martinelli, Santi Peris and Steve Sharpe for discussions.

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[2]The remaining "eye graphs" are those for which q ($\overline{q}$) is contracted with $\bar{s}$ (d) in eq. (1).